* Problem Sheet (2)
Q 1, 2, 3, 4, 5

11/5/2010

بسم الله الرحمن الرحيم

Quantitative Phys Lec (11)
Sheet 2 Solution

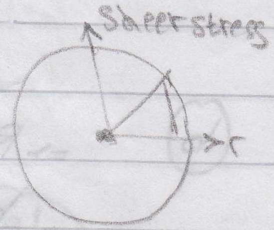
$\boxed{Q1}$:-

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma) = \frac{\Delta P}{L}$$

radial Co-ord

(a) Let: $\phi = r\sigma$

$$\therefore \frac{1}{r}\frac{\partial \phi}{\partial r} = \frac{\Delta P}{L} \Rightarrow \partial\phi = \frac{\Delta P}{L} r\,dr$$

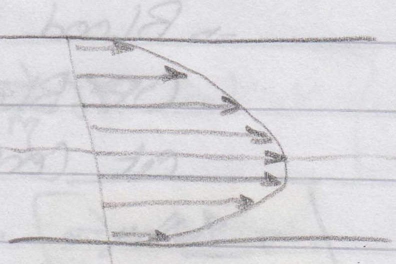


*Note:-
↳ As Layers of fluid differ in their speed, this will generate Shear Stress.

Forces bet. Layers

$\frac{\Delta F}{\Delta A}$ = Sheer stress

Area bet. Layers



$$\int_0^{\phi_0} \partial\phi = \frac{\Delta P}{L}\int_0^{r_0} r\,\partial r$$

$$\phi_0 = \frac{\Delta P}{L}\left[\frac{r^2}{2}\right]_0^{r_0}$$

$$\phi_0 = \frac{\Delta P}{L}\,\frac{r_0^2}{2}$$

$$\therefore\quad \phi_0 = r_0\,\tau_0$$

$$\therefore\quad r_0\,\tau_0 = \frac{\Delta P}{L}\,\frac{r_0^2}{2}$$

$$\tau_0 = \frac{\Delta P}{2L}\,r_0 \implies \boxed{\tau = \frac{\Delta P}{2L}\,r}$$

(b) => RBC's are moving Attached to each other, but at Certain Sheer Strees they will separate, => "Yeild Stress"

* Red Cell migration.
  => Blood Cells doesn't Attach to walls of Blood vessel, & they tends to be at Center of vessel
  Why..
  - => At walls => high Sheer Stress
  - => At Center => Low Sheer Stress.

$r = \frac{2L}{\Delta P} \sigma_y$ ⟹ Prove that

From (a):

$\because \sigma_y = \frac{\Delta P}{2L} R_y \Rightarrow \boxed{R_y = \frac{2L}{\Delta P} \sigma_y}$

Q 2 Show that:

Sheer stress vs Velocity Profile

→ "Laminar Flow Dynamic Flow" ⟹ Check lecture

★ Note: Sheer stress is given by:

$\sigma = \mu \frac{dV}{dr}$ → Velocity Profile

↳ Dynamic Viscosity $\frac{\text{Pascal}}{\text{sec}}$

→ Let μ be a Constant ⟹ Then Flow is newtonian

$\therefore \frac{dV}{dr} = \frac{\sigma}{\mu} \Rightarrow dV = \frac{\sigma}{\mu} dr$

$\therefore dV = \frac{1}{\mu} \frac{\Delta P}{2L} r\, dr \qquad \because \boxed{\sigma = \frac{\Delta P}{2L} r}$

$\because V(0) = V_{max}$

4

$$\int_{V_{max}}^{V} \partial V = \frac{1}{\eta} \frac{\Delta P}{2L} \int_{0}^{r} r\,dr$$

$$V - V_{max} = \frac{1}{\eta} \frac{\Delta P}{2L} \frac{r^2}{2}$$

$$\Rightarrow V = V_{max} + \frac{1}{\eta} \frac{\Delta P}{2L} \frac{r^2}{2}$$

$$\because V_{Boundary} = 0 = V(a)$$

$$\therefore 0 = V_{max} + \frac{1}{\eta} \frac{\Delta P}{2L} \frac{a^2}{2}$$

$$\therefore V_{max} = -\frac{1}{\eta} \frac{\Delta P}{2L} \frac{a^2}{2}$$

$$\therefore \frac{1}{\eta} \frac{\Delta P}{2L} = -\frac{2V_{max}}{a^2}$$

$$\therefore V = V_{max} - \frac{2V_{max}}{a^2} \cdot \frac{r^2}{2}$$

$$\therefore \boxed{V = V_{max} - V_{max} \frac{r^2}{a^2} = V_{max}\left(1 - \frac{r^2}{a^2}\right)}$$

3 given: $\boxed{\tau = \eta \frac{dV}{dr}}$, Newtonian → Constant η → Varying velocity.

(a) → From (Q2): $V = V_{max}\left(1 - \frac{r^2}{a^2}\right)$

$\therefore \frac{dV}{dr} = -2\frac{r}{a^2} V_{max}$

$\therefore \tau = \eta\left(-\frac{2r}{a^2} V_{max}\right)$

⟹ it is (-ve) As it is a Sheer stress opposing to Blood flow direction.

(b) The Dynamic Viscosity = 0.003 (Pa.s)

(I) $\because \tau = -\frac{2\eta V_{max}}{a^2} r$, $\because \eta = 0.003$ (Pa.s)

$\therefore V_{max} = 1 \frac{m}{sec}$, $a = 1\ cm = 0.01$ (m)

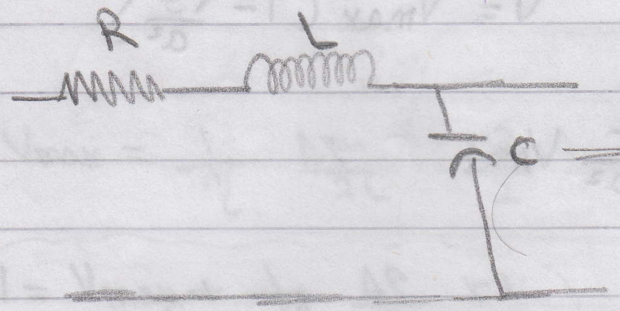
⟹ get $\boxed{\tau}$

(II) $|\tau_{max}| = \left|-\frac{2\eta V_{max}}{a^2} a\right|$

(6)

Q4

* Arterial Wall Segment:-

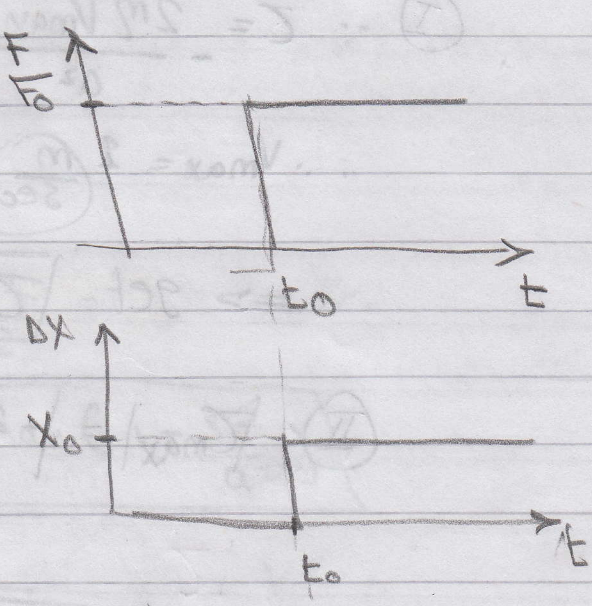


→ Represent effect of wall presence.

→ we let that there is no Branching in the Connection between Aorta & Iliac Blood vessles.

→ Capacitance → Compliance

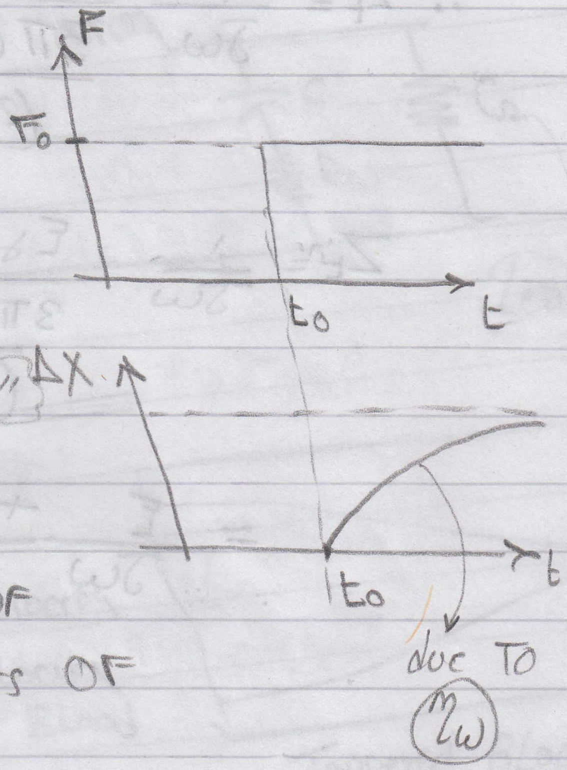
* elastic Behaviour. "Refere To hock Law"



Model of elasticity In spring.

★ Visco elasticity ::

→ we let that..
Blood vessel wall:
① elastic



Note: "Relaxation $\xrightarrow{Rep.}$ Viscoelasticity"

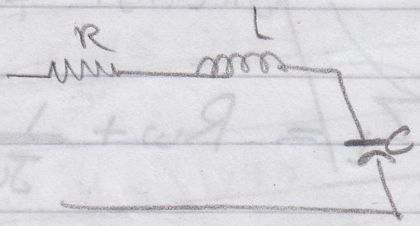
- E → young's modulus
  of avarage of E of
  various sublayers of
  Blood vessel.

E_{dyn} → Dyn. young's modulus

$$E_{dyn} = E + J\omega \eta_w$$

→ elasticity + viscosity
→ viscosity of wall

∴ ⓐ $Z_0 = \frac{1}{J\omega C}$ → in case of purely elastic



∴ $Z_t = \frac{1}{J\omega C_{dyn}}$

t → transverse

∵ C_{dyn} → is complex value that can't be found as physical element

$$\therefore Z_t = \frac{1}{j\omega}\;\frac{\quad}{\frac{3\pi(a+1)^2 r^2}{(2a+1)E_{dyn}}}$$

$$Z_t = \frac{1}{j\omega}\;\frac{E_{dyn}}{\frac{3\pi(a+1)^2 r^2}{(2a+1)}}$$

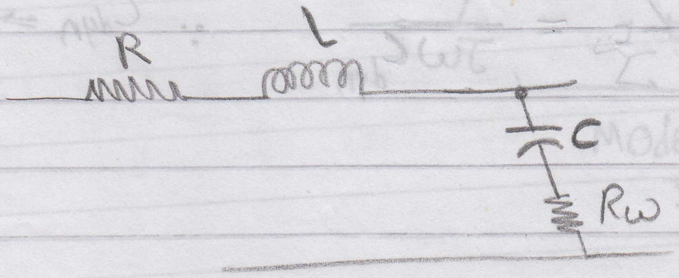
$$= \frac{\frac{E}{j\omega} + j\omega M_\omega}{\frac{3\pi(a+1)^2 r^2}{(2a+1)}}$$

$$\therefore Z_t = \frac{(2a+1)M_\omega}{3\pi(a+1)^2 r^2} + \frac{(2a+1)E}{j\omega\, 3\pi(a+1)^2 r^2}$$

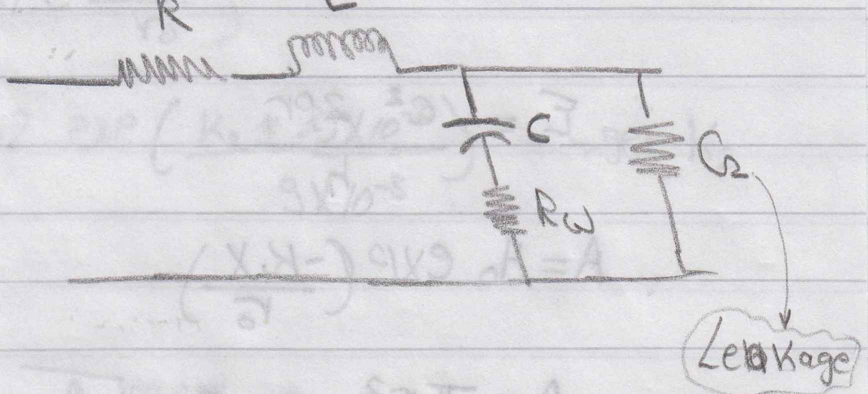
$$= \frac{(2a+1)}{3\pi(a+1)^2 r^2}\left[M_\omega + \frac{E}{j\omega}\right]$$

$$Z_t = R_\omega + \frac{1}{j\omega C}$$ --- impedance of Branch

Resistance + Capacitor } in series



(C) Leakage Rep.

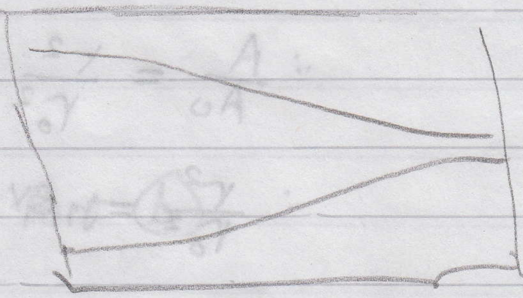


Q5

Phase velocity vs Blood velocity

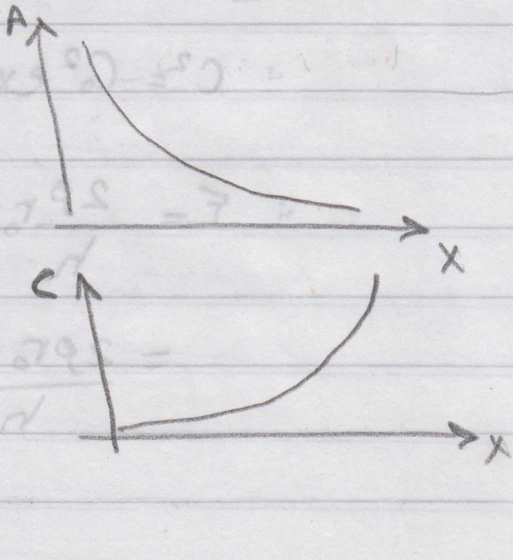
Blood velocity → velocity of Blood

Phase velocity → speed that mechanical wave Transmits within Blood vessel



Tappered Blood Vessel

* Geometric Tapering
  $A_0(x_0)$



(10)

$\rightarrow C^2 = \frac{Eh}{2\rho r}$ vessel wall Thickness

$\therefore E = \frac{C^2\, 2\rho r}{h}$

$\therefore A = A_0 \exp\left(\frac{-K_1 X}{r_0}\right)$

$\therefore A_0 = \pi r_0^2 \Rightarrow r = \sqrt{\frac{A}{\pi}}$

$\therefore \frac{A}{A_0} = \frac{r^2}{r_0^2}$

$\therefore \frac{r^2}{r_0^2} = \exp\left(\frac{-K_1 X}{r_0}\right)$

$\therefore \frac{r}{r_0} = \exp\left(\frac{-K_1 X}{2r_0}\right)$

$\therefore r = r_0 \exp\left(-\frac{K_1 X}{2r_0}\right)$

$\therefore C = C_0 \exp\left(\frac{+K_2 X}{r_0}\right)$

$\therefore C^2 = C_0^2 \exp\left(\frac{2K_2 X}{r_0}\right) \rightarrow$ given

$\therefore E = \frac{2\rho}{h} r_0 \exp\left(\frac{-K_1}{2r_0} X\right) \cdot C_0^2 \exp\left(\frac{2K_2 X}{r_0}\right)$

$= \frac{2\rho r_0 C_0^2}{h} \exp\left(\frac{2K_2 - K_1/2}{r_0} X\right)$

b

$\because \quad C = C_0 \exp\left(\frac{K_2 X}{r_0}\right)$

$\because \quad 7.5 = 4.2 \exp\left(\frac{K_2 * 39 \times 10^{-2}}{9 \times 10^{-3}}\right) \Longrightarrow$ get K_2

C given $K_{\cancel{1}} = 0.0367$, $\rho = 1.06 \ g/cm^3$

$h = 0.14 \ \cancel{km}m$

Find $E(x)$

Sol $\Rightarrow$ get K_2 From Part b

$\because \quad X = 39$ cm,

$R_0 = 9$ mm $\quad C_0 = 4.2$

$h = 0.14$ m

* Problem Sheet 2
Q 6,7,9,8,10,11,12

1

بسم الله الرحمن الرحيم

18/5/2016

Physiology Lec 12

Problem 6

$\boxed{A_{tot} = n \, A_{vessel}}$ ⟹ For every Case

n : No of vessels

Aorta : $A_{tot} = 2.5 \ cm^2$, $n=1$ $D = ???$

$$A_{vessel} = \frac{A_{tot}}{n} = \frac{2.5}{1} = 2.5 \ cm^2$$

$$2.5 = \frac{\pi D^2}{4} \longrightarrow D = 1.77$$

Cappillaries : $A_{tot} = 2500 \ cm$, $D = 8 \mu m$ $n = ??$

$$A_{vessel} = \frac{\pi D^2}{4} = \frac{\pi}{4}(8\times10^{-4})^2 = 5.265\times10^{-7} \ cm^2$$

$$n = \frac{A_{tot}}{A_{vessel}} = \frac{2500}{5.265\times10^{-7}} = 4.9\times10^9$$

* Vena Cavae:

$A_{tot} = 8 \ cm^2$, $n = 2$

$$A_{vessel} = \frac{A_{tot}}{n} = 4 \ cm^2 = \pi \frac{D^2}{4}$$

$D = 2.256$ cm

(2)

7) (a)

$$\therefore Q = \frac{\Delta P}{R} \qquad R = \frac{128 \mu L}{\pi D^4} = \frac{8 \mu L}{\pi r^4}$$

$$Q_1 = \frac{\Delta P}{R_1} , \quad Q_2 = \frac{\Delta P}{R_2}$$

$$\boxed{\frac{Q_2}{Q_1} = \frac{R_1}{R_2}}$$

$$R_1 = \frac{128 \mu L}{\pi D_1^4} , \quad R_2 = \frac{128 \mu L}{\pi D_2^4}$$

$$\frac{R_1}{R_2} = \left(\frac{D_2}{D_1}\right)^4$$

it is given that $\frac{Q_2}{Q_1} = \frac{1}{500}$

$$\therefore \frac{R_1}{R_2} = \frac{1}{500} = \left(\frac{D_2}{D_1}\right)^4 \longrightarrow \frac{D_2}{D_1} = 0.211$$

$$D_2 = 0.211 (30) = 6.33 \text{ (mm)}$$

3

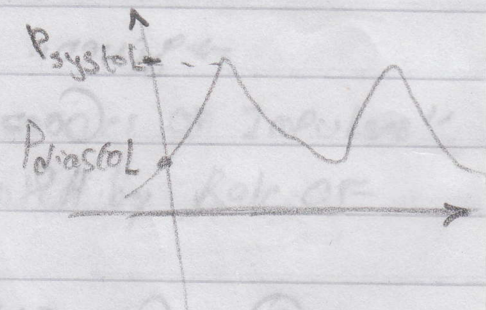
7b

→ Further Reduction in Local Blood Flow Can be Achived by the closor of the Pre capillary sphincter in that Location by the desired percentage (in that Case 50 percent).

★ Q8:-

⇒ given: - $Q = 5$ L/min.

- $P_a = 100$ mmHg



-

ⓐ $P_c = 30$ mmHg

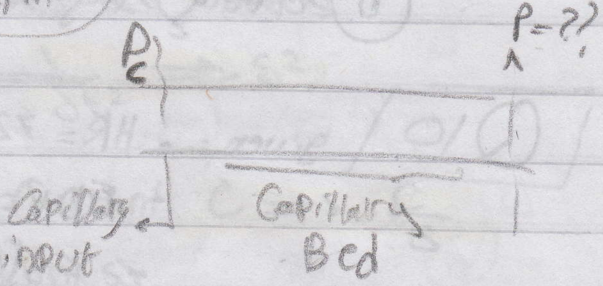
Pressure at Capillary Inlet

Calc. R

$$\rightarrow R_d = \frac{\Delta P}{Q} = \frac{100 - 30}{5} = 14 \ \frac{mmHg}{L/min}$$

Downstream Resistance

ⓑ given $R = 1 \ \frac{mmHg}{L/min}$



P_v , R_U → Up stream Resistance

Pressure at Capillary Outlet

$$\therefore R_U = \frac{\Delta P}{Q} = \frac{P_v - P_c}{Q} = \frac{P_v - \overset{???}{0}}{5 \ L/min} \ \frac{mmHg}{L/min}$$

↓ equal to 1

④

$\therefore P_V = 5$ (mmHg)

Q9 :-

W = weight ⟹ Kg , Co ⟹ mL/min , HR = BPM

⟹ given:

$Co = 224 \; w^{3/4}$

$HR = 229 \; w^{(-1/4)}$

ⓐ $\because CO = SV \cdot HR \Rightarrow SV = \frac{Co}{HR} = \frac{224 \, w^{3/4}}{229 \, w^{-1/4}}$ (mL) (CC) cubic cm

$= 0.978 \; W$

ⓑ Apply By values of w → 80, → 280

Q10 given: - HR = 72 BPM

- EDV = 140 mL

- ESV = 70 mL

ⓐ $CO = HR \times (EDV - ESV) = 5.04$ L/min

ⓑ From part A, we calculated

$CO|_1 = HR|_1 \cdot (EDV|_1 - ESV|_1)$ ⟹ At Rest

$CO|_2 = HR|_2 \, (EDV|_2 - ESV|_2)$

$= 3HR_1 \, (1.5 \, EDV|_2 - 0.5 \, ESV|_2) = 37.8 \; \frac{L}{min}$

Q11

Baro Recipetor:.

↳ Nerve endings with Special Functions

↳ it measures Pressure signal & Convert it into electrical Signal to be sent to Brain & Then Take Corresponding decision.

OF Neutral Firings "Frequancies OF Impulses"

=> Rate OF Firing is Represented by Rate OF Firing => $n(t)$.

$$p(t) \longrightarrow \boxed{\text{Pressure receptor}} \longrightarrow n(t)$$

=> SOL:

=> Assume initial Conditions by Zeros.

$$\frac{1}{\partial t} \Longrightarrow S \quad , \quad \frac{1}{\partial t^2} \Longrightarrow S^2$$

=> When There is Alone Constant $C \Longrightarrow \frac{C}{S}$

$$\Rightarrow \left[1+(T_1+T_2)S+T_1T_2S^2\right]n(s)$$
$$=\left\{(a_1+a_2)+\left[T_1(a_2+b_1)+T_2(a_1+b_1)\right]S + T_1T_2(b_1+b_2)S^2\right\}P(S)$$

⑥

$$\frac{N(s)}{P(s)} = \frac{(a_1+a_2) + [T_1(a_1+b_1) + T_2(a_1+b_2)]s + T_1T_2(b_1+b_2)s^2}{(1+T_1s)(1+T_2s)}$$

$\Rightarrow$ Consider the expression of the problem, This can be written As::

$$\therefore \frac{N(s)}{P(s)} = \frac{(a_1+b_1T_1s)(1+T_2s) + (a_2+b_2T_2s)(1+T_1s)}{(1+T_1s)(1+T_2s)}$$

$$= \frac{(a_1+a_2) + (b_1T_1 + a_1T_2 + b_2T_2 + a_2T_1)s + s^2(b_1T_1T_2 + b_2T_1T_2)}{(1+T_1s)(1+T_2s)}$$

ⓐ $T_1 = 5$ ms, $T_2 = 5$ sec

→ Compute Laplacian Transformation

$\therefore T_1 \ll T_2$

→ we notice that T_1 Can be neglicted.

$$\therefore N(s) = \left(a_1 + \frac{a_2+b_2T_2s}{1+T_2s}\right)P(s)$$

ⓑ given $P(t) = K\,u(t) \xrightarrow[\text{Transform}]{\text{Laplace}} P(s) = \frac{K}{s}$

$$\therefore N(s) = \left[a_1 + \left(\frac{a_2+b_2T_2s}{1+T_2s}\right)\right]\frac{K}{s}$$

$$= a_1\frac{K}{s} + \frac{(a_2+b_2T_2s)K}{s(1+T_2s)}$$

$$N(s) = \frac{a_1 K}{s} + \frac{K}{T_2} \, \frac{a_2 + b_2 T_2 s}{s(s + \frac{1}{T_2})}$$

$$= \frac{a_1 K}{s} + \frac{K}{T_2}\left[\frac{A_1}{s} + \frac{A_2}{s + \frac{1}{T_2}}\right]$$

$$\therefore A_1 = \left.\frac{a_2 + b_2 T_2 s}{s + \frac{1}{T_2}}\right|_{s=0} = a_2 T_2$$

$$\therefore A_2 = \left.\frac{a_2 + b_2 T_2 s}{s}\right|_{s=-1/T_2} = \frac{a_2 + b_2 \, T_2(-\frac{1}{T_2})}{-1/T_2}$$

$$= T_2 \, (b_2 - a_2)$$

$$\therefore N(s) = \frac{a_1 K}{s} + \frac{K}{\cancel{T_2}}\left[\frac{a_2 \cancel{T_2}}{s} + \frac{T_2(b_2 - a_2)}{s + \frac{1}{T_2}}\right]$$

$$\therefore \boxed{n(t) = a_1 K\, u(t) + K a_2\, u(t) + (b_2 - a_2)\, e^{-t/T_2}\, u(t)}$$

(8)

Q12

$$\Rightarrow Q(t) = C \frac{dP(t)}{dt} + \frac{P(t)}{R}$$ $\Rightarrow$ Representation of Charging & Releasing

$\Rightarrow$ Basal $\equiv$ Resting.

$$\rightarrow \underbrace{Q(t)}_{\text{Cardiac output}} = \underbrace{V_s}_{\text{Strock Volume}} \times \underbrace{H(t)}_{\text{Heart Rate}}$$

$$Q(t) = V_s \, H(t)$$

Eq(1):- $Q(t) = V_s \, H(t)$

$$\therefore Q(t) = V_s H_0 + V_s S_0 [P_0 - P(t)] - V_s S_1 \frac{dP}{dt}$$

$$= C \frac{dP(t)}{dt} + \frac{P(t)}{R} \quad (2)$$

(I)

$$\therefore \frac{dP}{dt}(C + V_s S_1) + \left[\frac{1}{R} + V_s S_0\right] P(t) = V_s H_0 + V_s S_0 P_0 = V_s (H_0 + S_0 P_0) \quad (3)$$

$\Rightarrow$

$$C_o \frac{dP}{dt} + G_{2o} P(t) = Q_o$$

→Note:- .when you solve it by Laplace, we should take Kare of initial Conditions.

given $P(o)$

$$\cancel{C_o [s P(s) + G_{2o} P(s) = \frac{Q_o}{s}}$$

$$\boxed{C_o [sP(s) - P(0)] + G_{2o} P(s) = \frac{Q_o}{s}}$$

$$\therefore (C_o s + G_{2o}) P(s) = C_o P(0) + \frac{Q_o}{s}$$

$$\therefore P(s) = \frac{P(0) . C_o}{C_o s + G_{2o}} + \frac{Q_o}{s(C_o s + G_{2o})}$$

$$= \frac{P(0)}{s + \underbrace{\frac{G_{2o}}{C_o}}_{\text{Time Constant.}}} + \frac{Q_o}{C_o s (s + \frac{G_{2o}}{C_o})}$$

Let $\frac{C_o}{G_{2o}} = T_o$

10

$$P(s) = \frac{P(0)}{s + \frac{1}{T_0}} + \frac{Q_0}{C_0} \cdot \frac{1}{s\left(s + \frac{1}{T_0}\right)}$$

$$\therefore P(s) = \frac{P_0}{s + \frac{1}{T_0}} + \frac{Q_0}{C_0}\left[\frac{A}{s} + \frac{B}{s + \frac{1}{T_0}}\right]$$

$$\therefore A = \frac{1}{s + \frac{1}{T_0}}\Big|_{s=0}, \quad B = \frac{1}{s}\Big|_{s = -1/T_0}$$

$$\therefore P(s) = \frac{P_0}{s + 1/T_0} + \frac{Q_0}{C_0}\left[\frac{T_0}{s} + \frac{-T_0}{s + \frac{1}{T_0}}\right]$$

$$P(t) = P_0 e^{-t/T_0} + \underbrace{\frac{\overset{\text{Current}}{Q_0} T_0}{\underset{\text{Capacitance}}{C_0}}\left[1 - e^{-\overset{\text{time}}{t}/T_0}\right]}_{\text{Pressure = Voltage}}$$

$$\therefore T_0 = \frac{C_0}{G_{20}} = \frac{C + V_s S_1}{\frac{1}{R} + V_s S_0} \Rightarrow \boxed{a = \frac{1}{T_0} = \frac{\frac{1}{R} + V_s S_0}{C + V_s S_1}} \text{ given}$$

$$\boxed{\frac{Q_0 T_0}{C_0} = \frac{\left[V_s(N_0 + S_0 P_0)\right]}{\cancel{C + V_s S_1}} \cdot \frac{\cancel{C + V_s S_1}}{\frac{1}{R} + V_s S_0} = P_1}$$