هيدروليك
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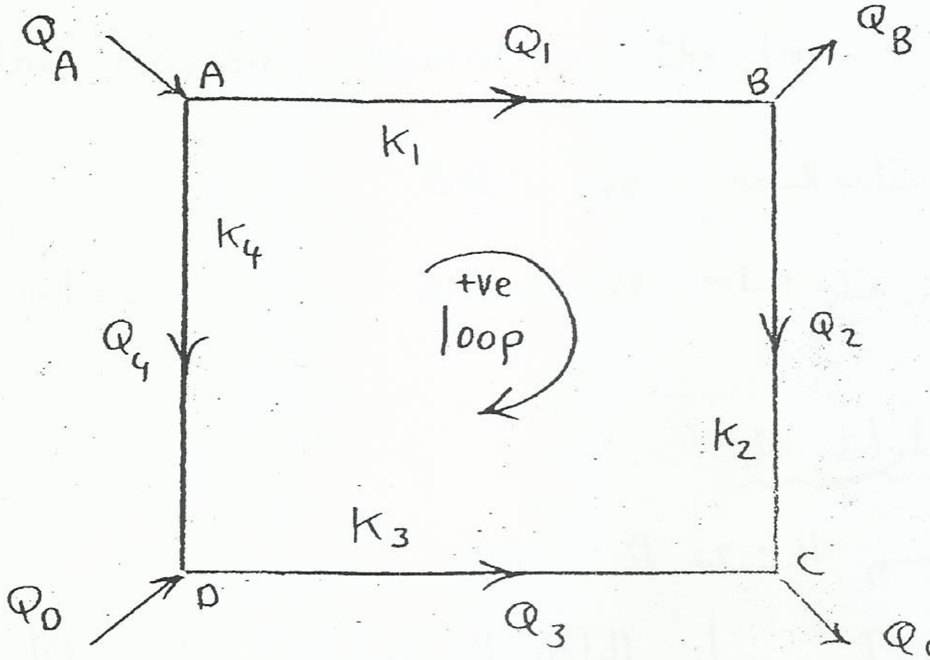
# 1 Pipe Networks

## Hardy cross method




For any loop

[1] $\sum Q_{in} = \sum Q_{out}$

e.g// $Q_A + Q_D = Q_B + Q_C$

[2] $\sum Q$ at any junction $= 0$

e.g// $Q_A = Q_1 + Q_4$

e.g// $Q_1 = Q_B + Q_2$

[3] $\sum h_L$ around any loop $= 0$

e.g// $h_{L_{AB}} + h_{L_{BC}} + h_{L_{CD}} + h_{L_{DA}} = 0$

حفظ

القانون المستخدم

4

$$\Delta Q = \frac{-\sum K Q^{n}}{n \sum \left| K Q^{n-1} \right|}$$

where $\Delta Q$ is the correction required in the loop

$n = 2$ darcy — $\Delta Q = +ve$ مع عقارب الساعة

$n = 1.85$ Hazen william — $\Delta Q = -ve$ عكس عقارب الساعة

خطوات الحل

١- نقسم الشبكة إلى loops

٢- التأكد من أن الداخل إلى الشبكة يساوى الخارج منها

٣- فرض Q فى كل فرع من أفرع الشبكة (المواسير) مع مراعاة أن $\sum Q = 0$ عند أى junction

٤- حساب $\Delta Q$ من القانون السابق

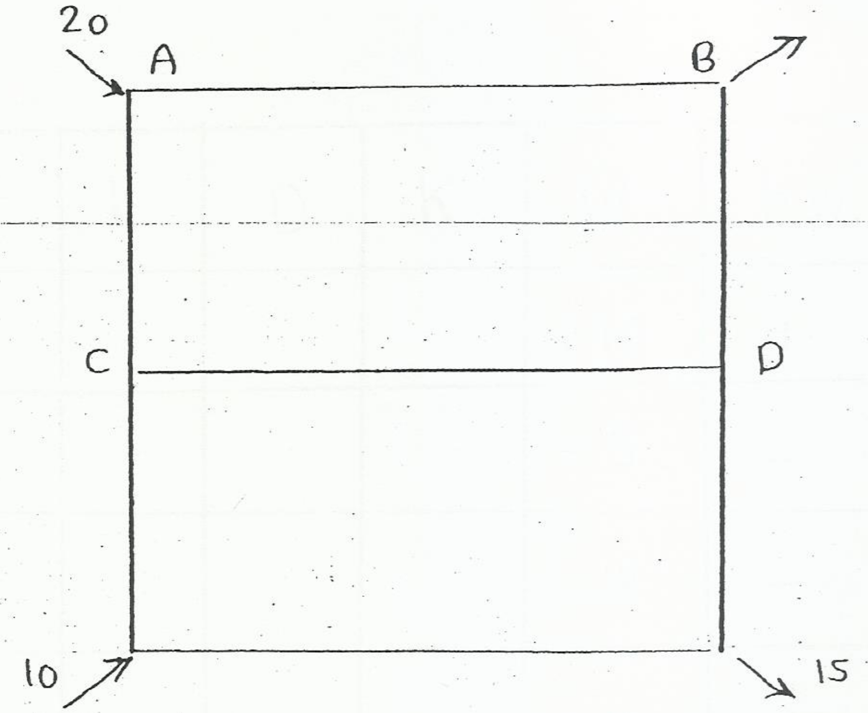
٥- إذا كانت $\Delta Q$ تساوى صفر ⇐ الفرض صحيح

إذا كانت $\Delta Q$ لا تساوى صفر ⇐ نضيف $\Delta Q$ بإشارتها على Q الموجودة فى المواسير

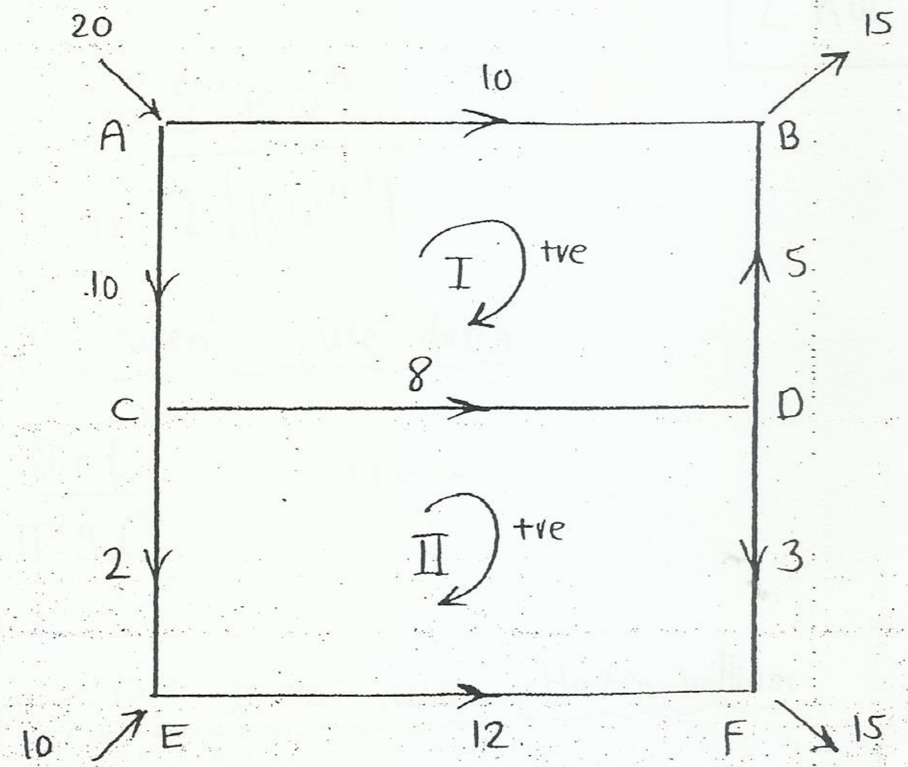
٦- نرسم الشبكة مرة أخرى بعد التصحيح (بعد إضافة $\Delta Q$) مع مراعاة أكثر من تصحيح لبعض المواسير [مع عقارب الساعة +ve ↻]

٧- حساب $\Delta Q$ جديدة فى المواسير وهكذا حتى تقترب من الصفر أو الاكتفاء بـ 2 trials

Example (1)




Solution




$\sum Q_{in} = \sum Q_{out}$ ✓

$\sum Q$ at any junction $= 0$ ✓

loop 1

| Pipe | F | L | D | K | Q | $KQ^n$ | $\lvert KQ^{n-1} \rvert$ |
|---|---|---|---|---|---|---|---|
| AB | | | | | +10 | + | + |
| BD | | | | | −5 | − | + |
| DC | | | | | −8 | − | + |
| CA | | | | | −10 | − | + |
| | | | | | | $\Sigma KQ^n$ | $\Sigma KQ^{n-1}$ |

$$\Delta Q = \frac{-\Sigma KQ^n}{n \, \Sigma \lvert KQ^{n-1} \rvert}$$

If F is given use darcy

$$K = \frac{8FL}{\pi^2 g d^5} \qquad n = 2$$

If $C_{HW}$ is given use Hazen william

$$K = \frac{10.7\,L}{d^{4.86} C^{1.85}} \quad \text{SI} \qquad K = \frac{4.73\,L}{d^{4.86} C^{1.85}} \quad \text{Eng}$$

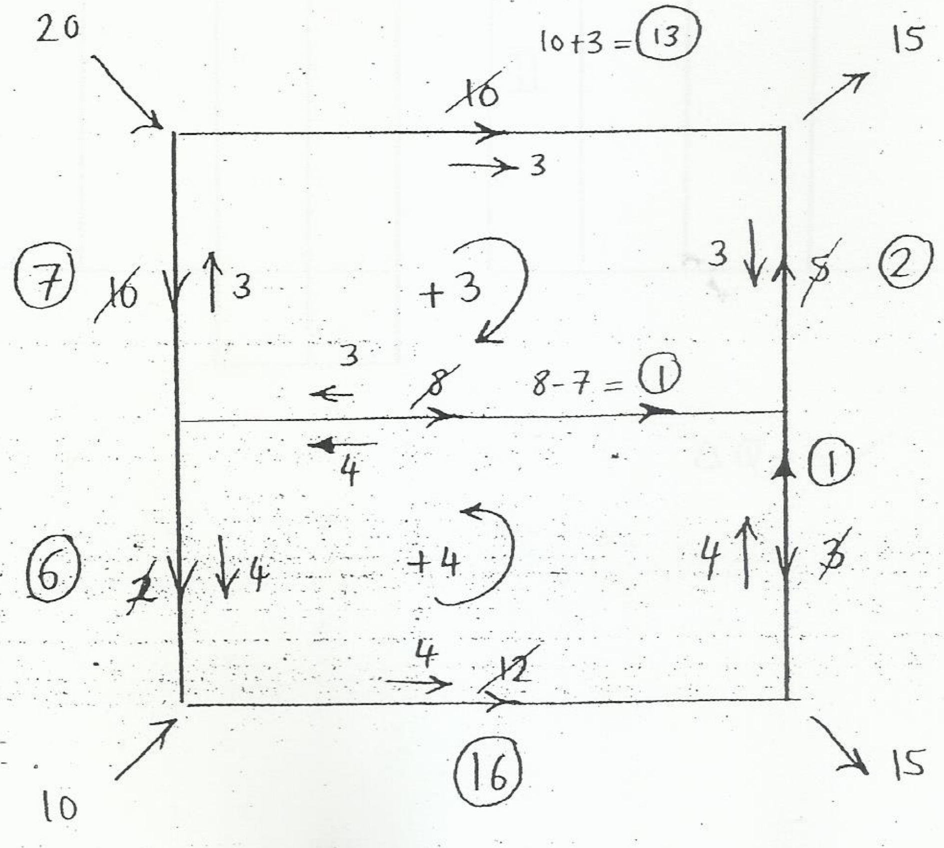
$n = 1.85$

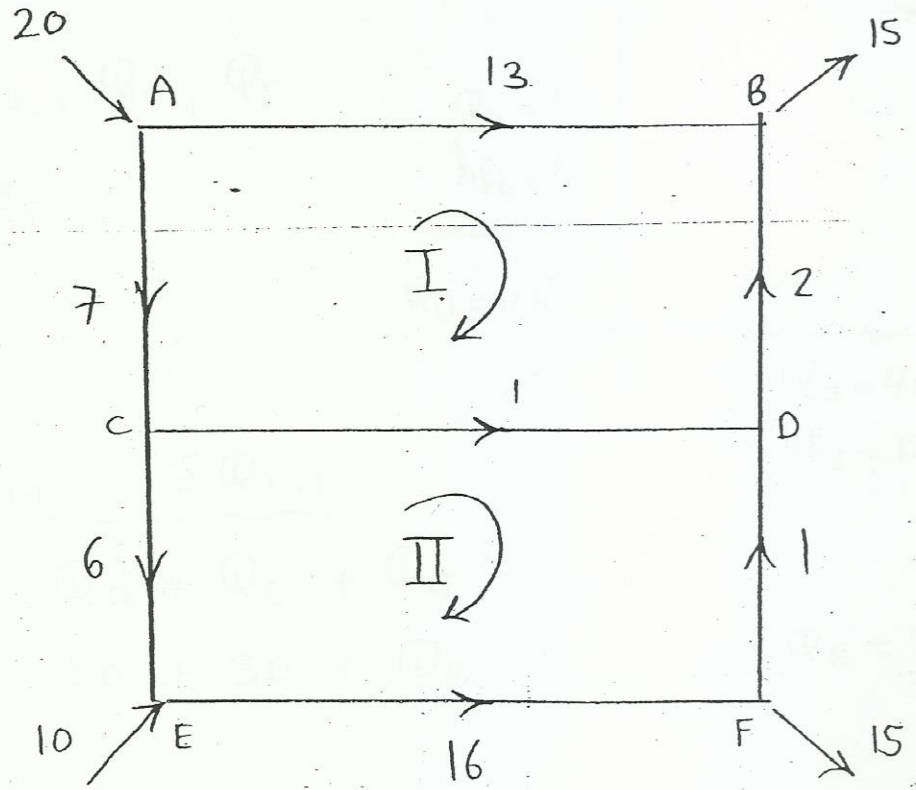
loop 2

| Pipe | F | L | D | K | Q | $KQ^n$ | $\lvert KQ^{n-1}\rvert$ |
|---|---|---|---|---|---|---|---|
| CD | | | | | +8 | + | + |
| DF | | | | | +3 | + | + |
| FE | | | | | −12 | − | + |
| CE | | | | | −2 | − | + |
| | | | | | | $\Sigma KQ^n$ | $\Sigma\lvert KQ^{n-1}\rvert$ |

$$\Delta Q_2 = \frac{-\Sigma KQ^n}{n\,\Sigma\lvert KQ^{n-1}\rvert}$$

Say $\Delta Q_1 = +3$ , $\Delta Q_2 = -4$ تصحيح الشبكة






2nd trial

| loop | pipe | K | Q | $KQ^n$ | $\lvert KQ^{n-1} \rvert$ |
|---|---|---|---|---|---|
| I | | | | | |
| | | | | | |

$\Delta Q_1 = \checkmark$

| loop | pipe | K | Q | $KQ^n$ | $\lvert KQ^{n-1} \rvert$ |
|---|---|---|---|---|---|
| II | | | | | |
| | | | | | |

$\Delta Q_2 = \checkmark$

recorrect

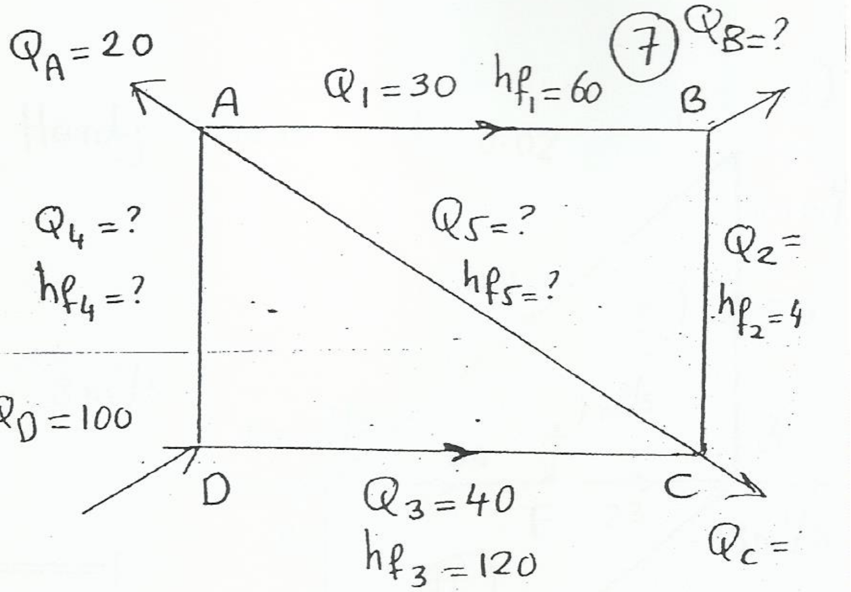
Ex. (2)

**Find**

$Q_B$, $Q_2$, $Q_4$, $Q_5$

$h_{f4}$, $h_{f5}$






[1] $\Sigma Q_{in} = \Sigma Q_{out}$

$Q_D = Q_a + Q_C + Q_B$

$100 = 20 + 30 + Q_B$ $\Rightarrow Q_B = 50$

[2] $\Sigma Q$ around any junction $= 0$

Junction D: $Q_D = Q_3 + Q_4$

$100 = 40 + Q_4$ $\Rightarrow Q_4 = 60$

Junction A: $Q_4 = Q_A + Q_1 + Q_5$

$60 = 20 + 30 + Q_5$ $\Rightarrow Q_5 = 10$

Junction C $\Rightarrow Q_2 = 20$

check Junction B $30 + 20 = 50$

[3] $\Sigma$ head loss around any loop $= 0$

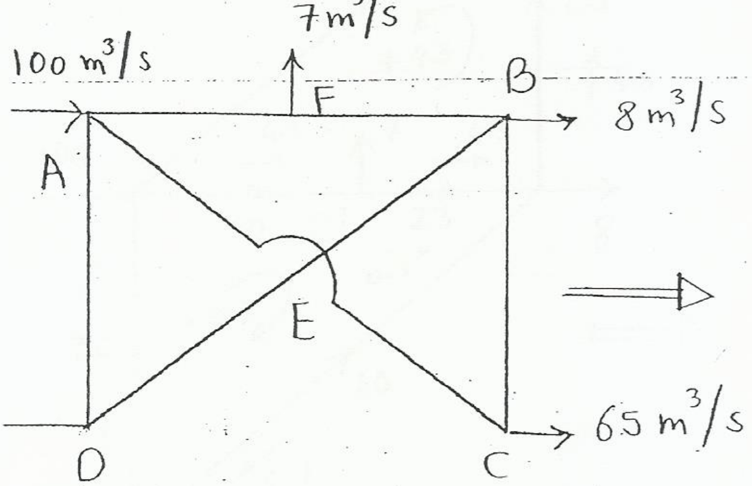
loop ABC $h_{f1} - h_{f2} - h_{f5} = 0$

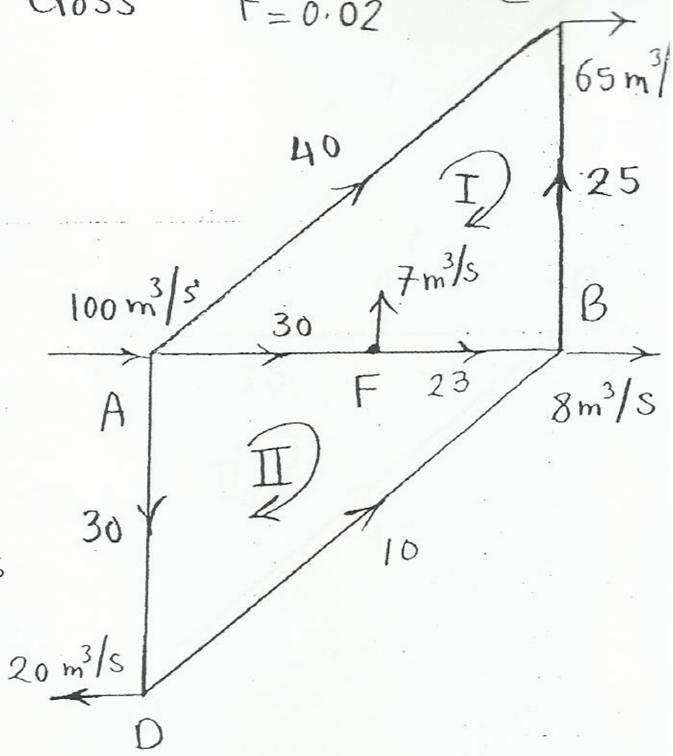
$60 - 40 - h_{f5} = 0$ $\Rightarrow h_{f5} = 20$

loop ADC $h_{f4} + h_{f5} - h_{f3} = 0$

$h_{f4} + 20 - 120 = 0$ $\Rightarrow h_{f4} = 100$

Use 2 steps of Hardy Cross F=0.02




## 1st Trial

### Loop I

| Pipe | D | L | F | K | Q | $KQ^2$ | $K\|Q\|$ |
|---|---|---|---|---|---|---|---|
| AC | 0.3 | 1000 | 0.02 | 680 | +40 | +1088090 | +27202 |
| BC | 0.3 | 600 | 0.02 | 408 | -25 | -255021 | +10200 |
| AF | 0.3 | 400 | 0.02 | 272 | -30 | -244820 | +8160 |
| FB | 0.3 | 400 | 0.02 | 272 | -23 | -143900 | +6256 |
| | | | | | | 444349 | 51818 |

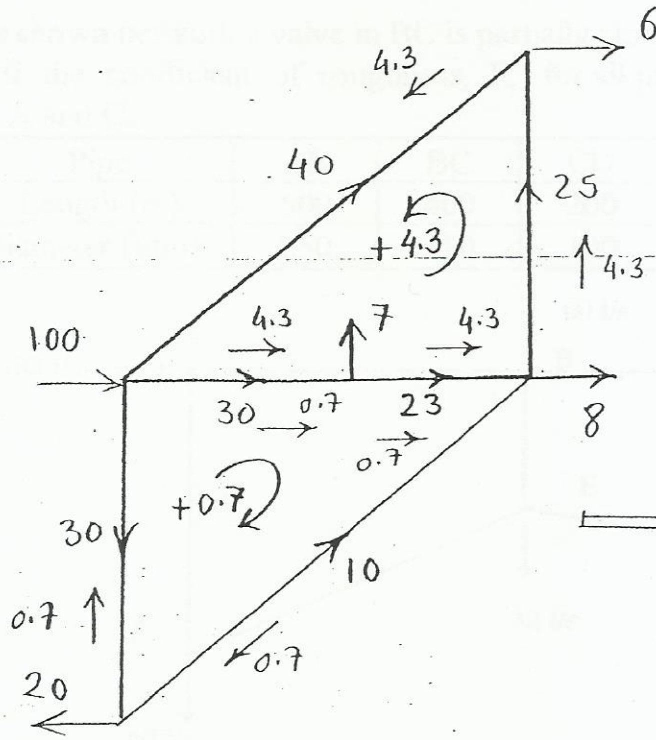
$$\Delta Q_I = -\frac{\Sigma KQ^2}{2\Sigma K|Q|} = -4.3$$

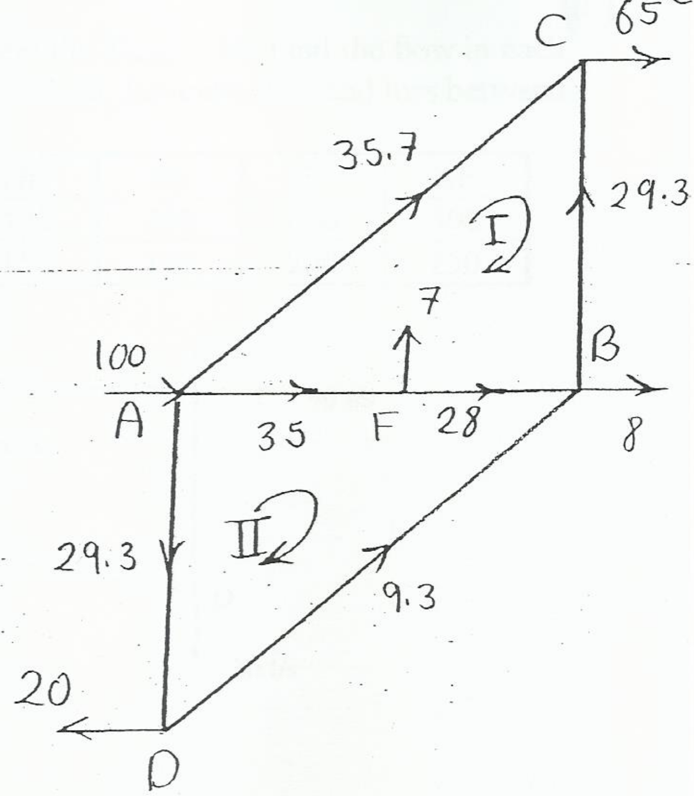
### Loop II

| Pipe | D | L | F | K | Q | $KQ^2$ | $K\|Q\|$ |
|---|---|---|---|---|---|---|---|
| AD | 0.3 | 600 | 0.02 | 408 | -30 | -367230 | +12241 |
| DB | 0.3 | 1000 | 0.02 | 680 | -10 | -68005 | +6800 |
| AF | 0.3 | 400 | 0.02 | 272 | +30 | +244820 | +8160 |
| FB | 0.3 | 400 | 0.02 | 272 | +23 | +143900 | +6256 |
| | | | | | | -46515 | 33457 |

$\Delta Q_{II} = +0.7$

(A)



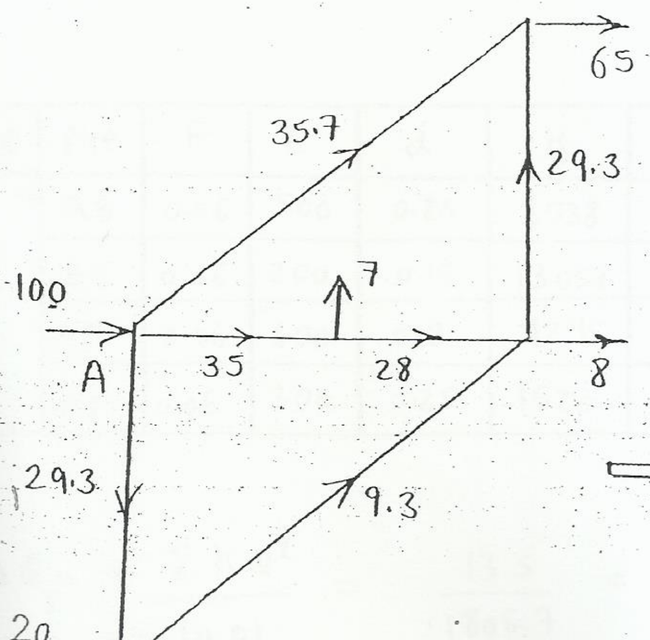

## 2nd Trial

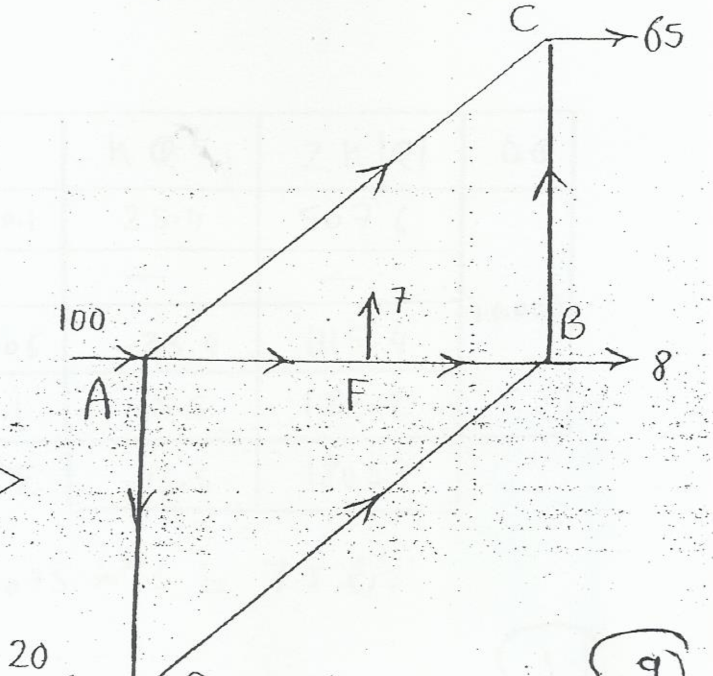
| loop | Pipe | K | Q | $KQ^2$ | $K\|Q\|$ |
|---|---|---|---|---|---|
| I | AC | 680 | +35.7 | | |
| | BC | 408 | -29.3 | | |
| | AF | 272 | -35 | | |
| | FB | 272 | -28 | | |
| | | | | | |

$\Delta Q_I =$

| loop | Pipe | K | Q | $KQ^2$ | $K\|Q\|$ |
|---|---|---|---|---|---|
| II | AD | 408 | -29.3 | | |
| | DB | 680 | -9.3 | | |
| | AF | 272 | +35 | | |
| | FB | 272 | +28 | | |
| | | | | | |

$\Delta Q_{II} =$



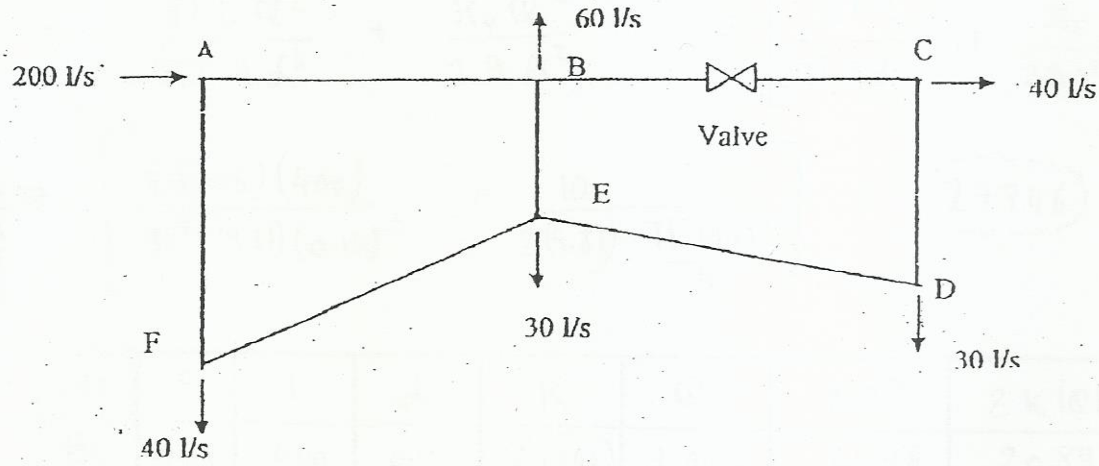



Ex: (3)

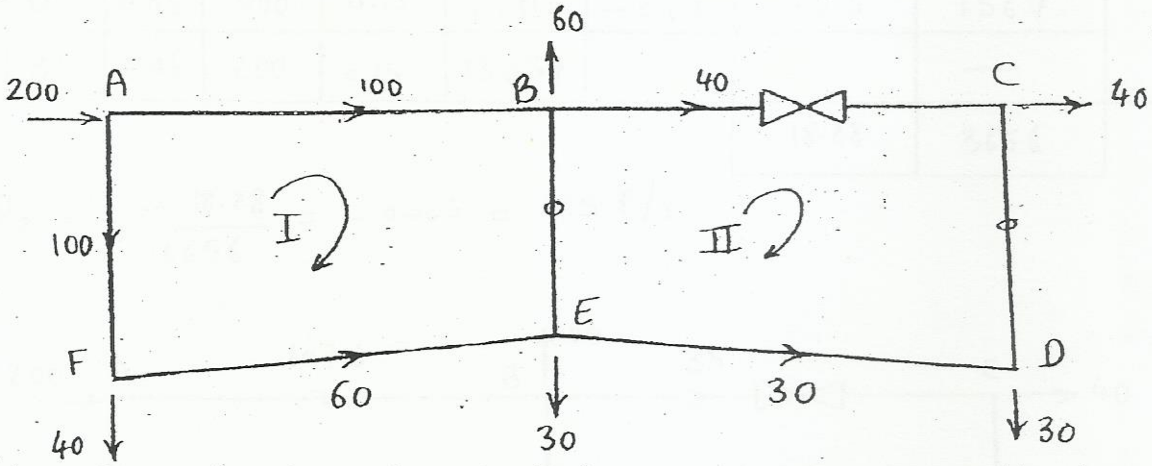


In the shown network a valve in BC is partially closed and the $K_{valve} = 10$. Find the flow in each pipe if the coefficient of roughness, F, for all pipes is 0.06. Estimate the head loss between point A and C.

| Pipe | AB | BC | CD | DE | BE | EF | AF |
|---|---|---|---|---|---|---|---|
| Length (m) | 500 | 400 | 200 | 400 | 200 | 600 | 300 |
| Diameter (mm) | 250 | 150 | 100 | 150 | 150 | 200 | 250 |




**Solution**




| loop | Pipe | F | L | d | K | Q | $KQ^2$ | $2K\lvert Q\rvert$ | ΔQ |
|---|---|---|---|---|---|---|---|---|---|
| | AB | 0.06 | 500 | 0.25 | 2538 | + 0.1 | 25.4 | 507.6 | |
| I | BE | 0.06 | 200 | 0.15 | 13057 | — | — | — | +0.0075 |
| | EF | 0.06 | 600 | 0.2 | 9295 | −0.06 | −33.4 | 1115.4 | |
| | AF | 0.06 | 300 | 0.25 | 1523 | −0.1 | −5.5 | 182.7 | |
| | | | | | | Σ | −13.5 | 1805.7 | |

$$\Delta Q_1 = -\frac{\Sigma KQ^2}{2\Sigma |KQ|} = \frac{13.5}{1805.7} = 0.0075 \ m^3/s = 7.5 \ l/s$$

For Pipe BC

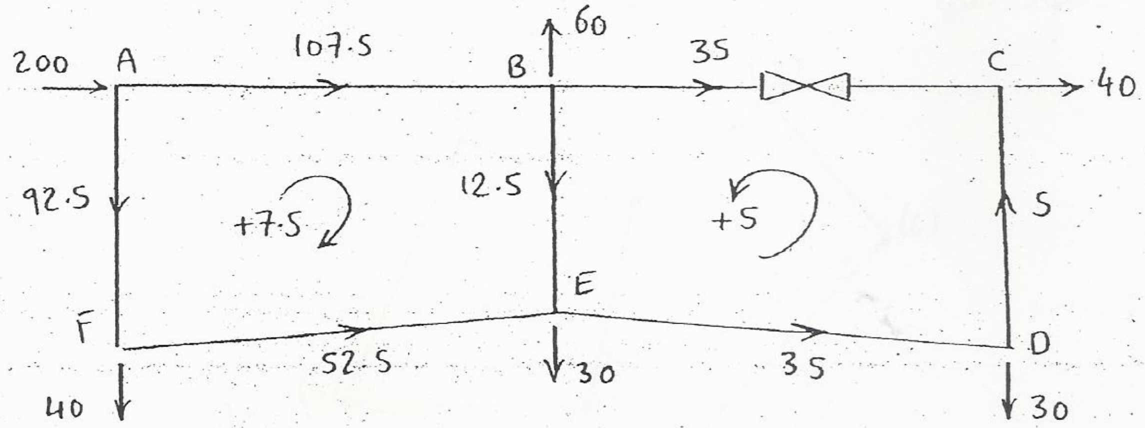
$$h_L = \frac{8FLQ^2}{\pi^2 g d^5} + \frac{K_v V^2}{2g}$$

$$= \frac{8FLQ^2}{\pi^2 g d^5} + \frac{K_v Q^2}{2gA^2} = \overbrace{\left[\frac{8FL}{\pi^2 g d^5} + \frac{K_v}{2gA^2}\right]}^{K_{BC}} Q^2$$

$$\therefore K_{BC} = \left[\frac{8(0.06)(400)}{\pi^2 (9.81)(0.15)^5} + \frac{10}{2(9.81)\left(\frac{\pi(0.15)^2}{4}\right)^2}\right] = 27746$$

| loop | Pipe | F | L | d | K | Q | $KQ^2$ | $2K\|Q\|$ | $\Delta Q$ |
|---|---|---|---|---|---|---|---|---|---|
| II | BC | 0.06 | 400 | 0.15 | 27746 | +0.04 | 41.78 | 2089 | -0.005 |
| | CD | 0.06 | 200 | 0.1 | 99152 | — | — | — | |
| | DE | 0.06 | 400 | 0.15 | 26114 | -0.03 | -23.5 | 1567 | |
| | BE | 0.06 | 200 | 0.15 | 13057 | — | — | — | |
| | | | | | | | 18.28 | 3656 | |

$$\Delta Q_2 = -\frac{18.28}{3656} = -0.005 = -5\ l/s$$




$$h_{L_{A\to C}} = h_{L_{AB}} + h_{L_{BC}}$$

$$= K_{AB} Q_{AB}^2 + K_{BC} Q_{BC}^2$$

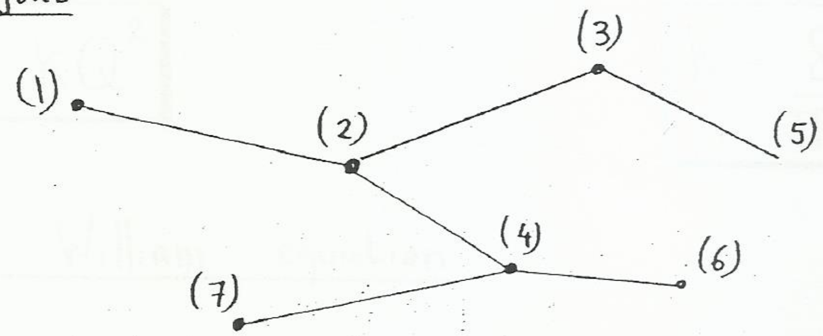
$$= 2538(0.1075)^2 + 27746(0.035)^2 = \underline{\underline{63.3\ m}}$$

# Pipe Network Analysis

## 1- Reliability إعتمادية

A network can be a tree network or a looped one.

### a) Tree Layout

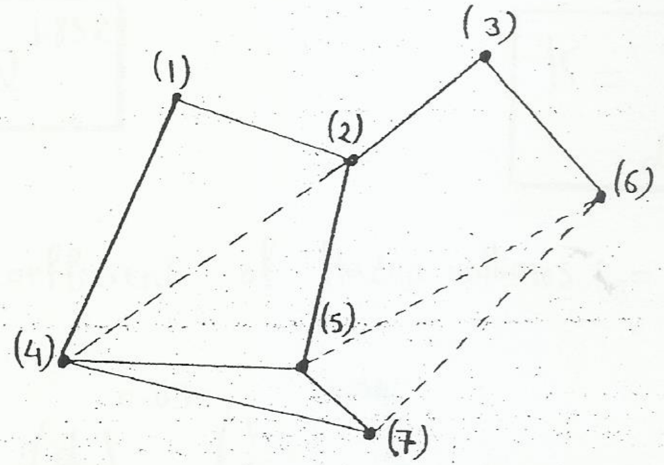



Tree layout is used in rural (ريف) areas or for pipe irrigation

The tree pipes are the minimum no. of pipes required

### b) Loop layout

- Loops are needed for reliability purposes حتى يمكن الاعتماد عليها




## 2- Analysis, Design and Optimization

The design problems are solved by trial and error

Optimization is to design with the minimum cost of network

The cost includes both capital and running costs

## Friction losses eqns used in solving pipe networks

### 1- Darcy - Weisbach equation

$$h_f = \frac{FLV^2}{2gd} = \frac{8FLQ^2}{\pi^2 g d^5}$$

$$\boxed{h_f = KQ^2} \qquad \boxed{K = \frac{8FL}{\pi^2 g d^5}}$$

### 2- Hazen - William equation

SI units

$$V = 0.85\, C \left(\frac{d}{4}\right)^{0.63} \left(\frac{h_L}{L}\right)^{0.54}$$

$$h_L = \frac{10.7\, L\, Q^{1.852}}{d^{4.87}\, C^{1.852}}$$

$$\boxed{h_L = KQ^{1.852}} \qquad \boxed{K = \frac{10.7\, L}{d^{4.87}\, C^{1.852}}}$$

C: roughness coefficient of Hazen williams = 110 → 140 for Smooth pipes

Eng units

$$V = 1.32\, C \left(\frac{d}{4}\right)^{0.63} \left(\frac{h_L}{L}\right)^{0.54}$$

$$h_L = \frac{4.73\, L\, Q^{1.852}}{d^{4.87}\, C^{1.852}}$$

$$\boxed{h_L = KQ^{1.852}} \qquad \boxed{K = \frac{4.73\, L}{d^{4.87}\, C^{1.852}}}$$

## Analysis Techniques :-

- They are the techniques used to solve the pipe networks

The techniques are classified according to the Variables used as unknowns to solve the network.

- There are 4 analysis techniques

1) Q - system ✓
2) H - system ✓
3) ΔQ - System } مش عليك
4) ΔH - system }

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### [1] Q-System [pipes flows as unknowns]

* No. of equations = (No. of nodes − 1) + No. of loops

where Node equ. : $\Sigma Q = 0$ at the node [ex.- $Q_1 + Q_2 - Q_3 = 0$]

Loop equ. : $\Sigma h_L = 0$ at the loop ∴ $\Sigma K Q^n = 0$

[ex.:- $K_1 Q_1^n + K_2 Q_2^n = 0$] → Q is unknown → [get Q]

### [2] H-System [Heads at nodes as unknowns]

* No. of equations = No. of nodes − 1

where Node equ. :- $\Sigma Q = 0$ at each node

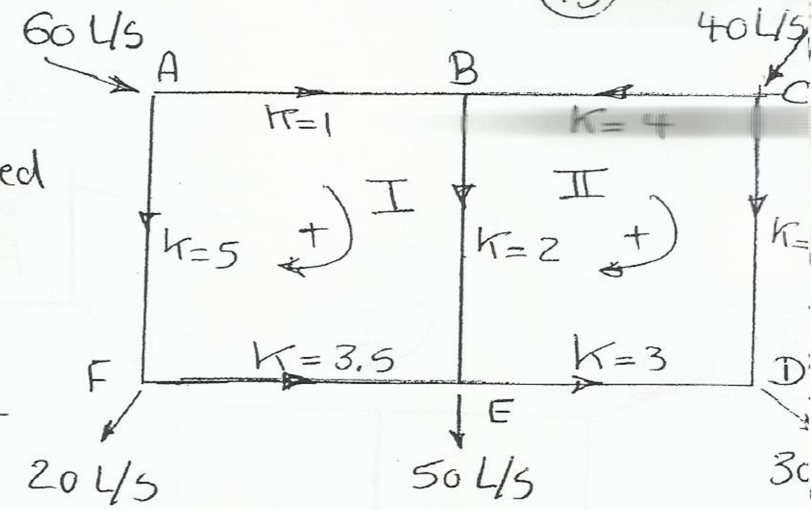
but $h_L = H_1 - H_2 = K_{1-2} Q_{1-2}^n \Rightarrow Q_{1-2} = \left[\frac{H_1 - H_2}{K_{1-2}}\right]^{1/n}$

∴ Node equ. : $\sum \left(\frac{H_a - H_b}{K_{ab}}\right)^{1/n} = 0$

ex:- $Q_1 + Q_2 = 0 \Rightarrow \left[\frac{H_1 - H_2}{K}\right]^{1/n} + \left[\frac{H_1 - H_3}{K}\right]^{1/n} = 0$ → [get H]

## Ex: (4)

Write down the Equations required to solve this network




### a) Pipe Flows (Q-system)

$$60 - Q_{ab} - Q_{af} = 0$$

$$Q_{ab} + Q_{bc} - Q_{be} = 0$$

$$40 - Q_{bc} - Q_{cd} = 0$$

$$Q_{cd} + Q_{de} - 30 = 0$$

$$Q_{be} + Q_{ef} - Q_{de} - 50 = 0$$

} Nodes equations

No. of equ. = Nodes - 
= 6 - 1 = 5

$$1 \times Q_{ab}^{n} + 2Q_{be}^{n} - 3.5Q_{ef}^{n} - 5Q_{af} = 0$$ → loop I equation

$$-4Q_{bc}^{n} + 1.5Q_{cd}^{n} - 3Q_{de}^{n} - 2Q_{be}^{n} = 0$$ → loop II equation

### b) Heads at nodes (H-system)

$h_L = KQ^n \rightarrow Q = \left(\frac{h_L}{K}\right.$

$$\left(\frac{H_a - H_b}{1}\right)^{1/n} - \left(\frac{H_a - H_f}{5}\right)^{1/n} + 60 = 0$$

$$\left(\frac{H_a - H_b}{1}\right)^{1/n} + \left(\frac{H_c - H_b}{4}\right)^{1/n} - \left(\frac{H_b - H_e}{2}\right)^{1/n} = 0$$

$$40 - \left(\frac{H_c - H_b}{4}\right)^{1/n} - \left(\frac{H_c - H_d}{1.5}\right)^{1/n} = 0$$

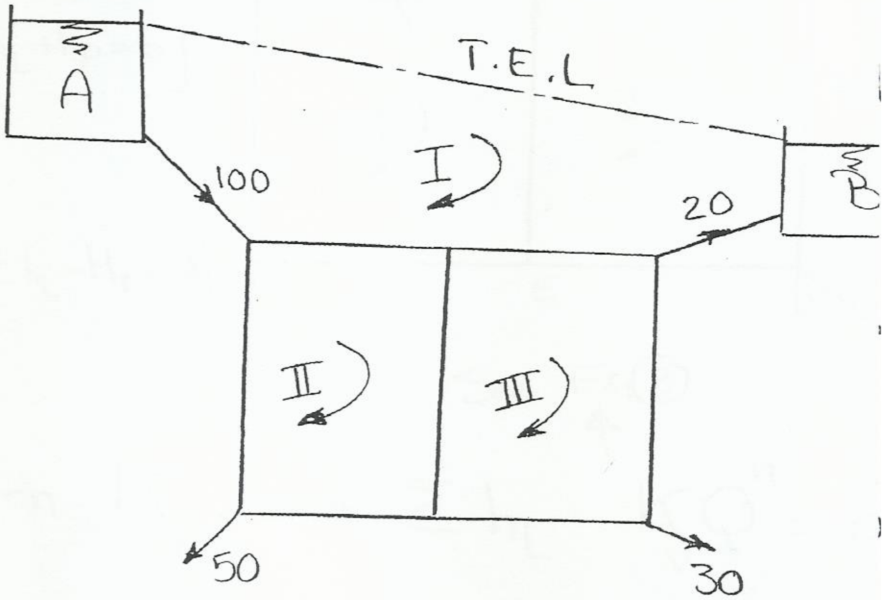
$$\left(\frac{H_c - H_d}{1.5}\right)^{1/n} + \left(\frac{H_e - H_d}{3}\right)^{1/n} - 30 = 0$$

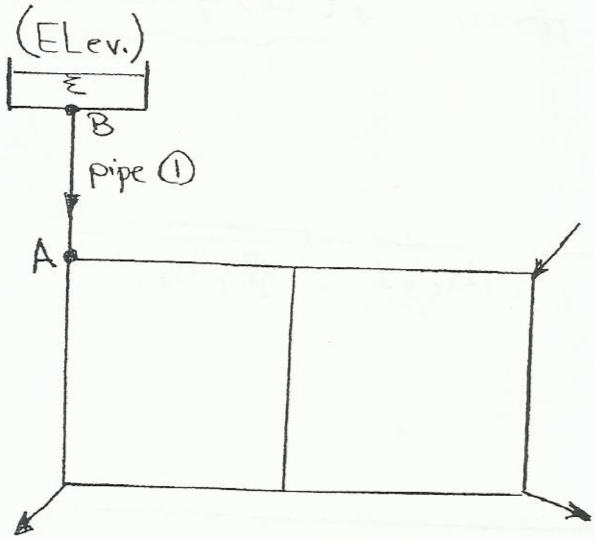
$$\left(\frac{H_b - H_e}{2}\right)^{1/n} - \left(\frac{H_e - H_d}{3}\right)^{1/n} - \left(\frac{H_f - H_e}{3.5}\right)^{1/n} - 50 = 0$$

} No. of 
= Node
= 5

# Special Cases :

* Consider an outter loop bet (A), (B)

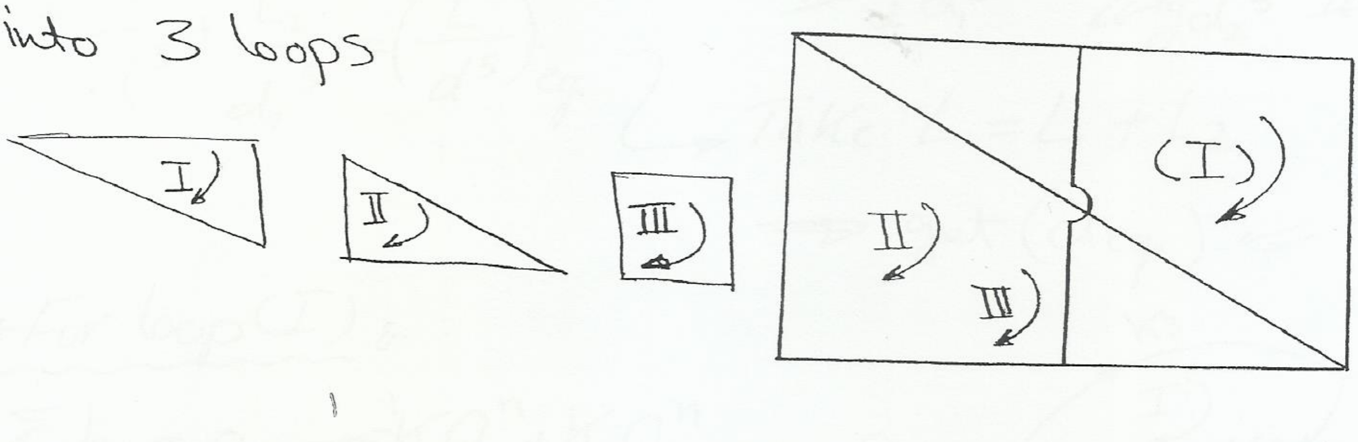


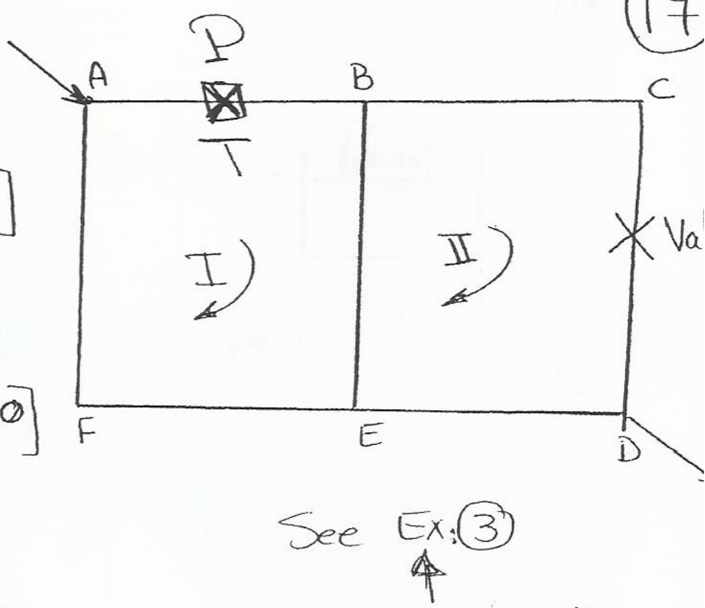

* if given tank elev. only (K of pipe① is not given & Don't take pipe ① in network & ($H_A$ = Elev.)
* if given (K) & (Elev.)

∴ Take pipe ① in equations where $h_{L_{BA}} = K_1 Q_{BA}^n$

* Devide this network into 3 loops



* **For loop(I):**



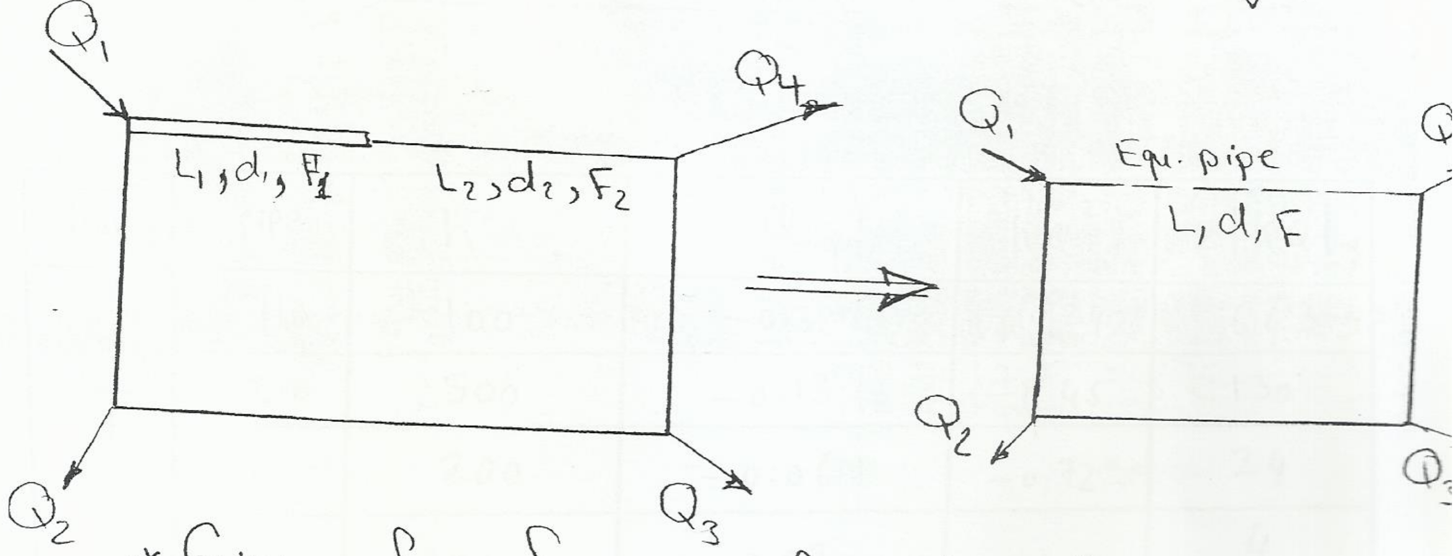

in Case of pump $\left[\sum h_L + h_p = 0\right]$

$$\rightarrow \Delta Q = \frac{-(\sum KQ^n + H_p)}{n\sum |KQ^{n-1}|}$$

in case of Turbine $\left[\sum h_L - H_T = 0\right]$

$$\rightarrow \Delta Q = \frac{-(\sum KQ^n - H_T)}{n\sum |KQ^{n-1}|}$$

* **For loop(II):** given $K_{valve}$ $\rightarrow \sum h_L - K_V Q^n = 0$




* Given $f_1 = f_2$ $\therefore f_{eq} = f_1 = f_2$

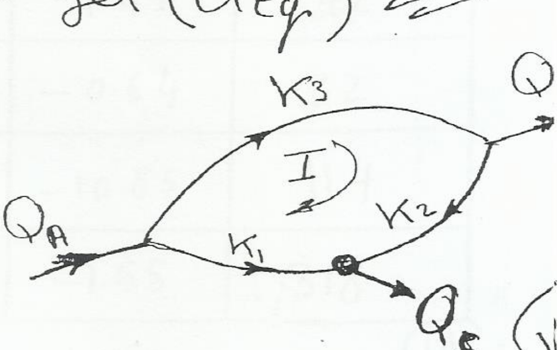
$$h_{f_1} + h_{f_2} = h_{f_{eq}} \Rightarrow \frac{8f_1L_1Q^2}{\pi^2 g d_1^5} + \frac{8f_2L_2Q^2}{\pi^2 g d_2^5} = \frac{8fL\ldots}{\pi^2 g\ldots}$$

$$\therefore \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} = \left(\frac{L}{d^5}\right)_{eq}$$

→ Take $L = L_1 + L_2$

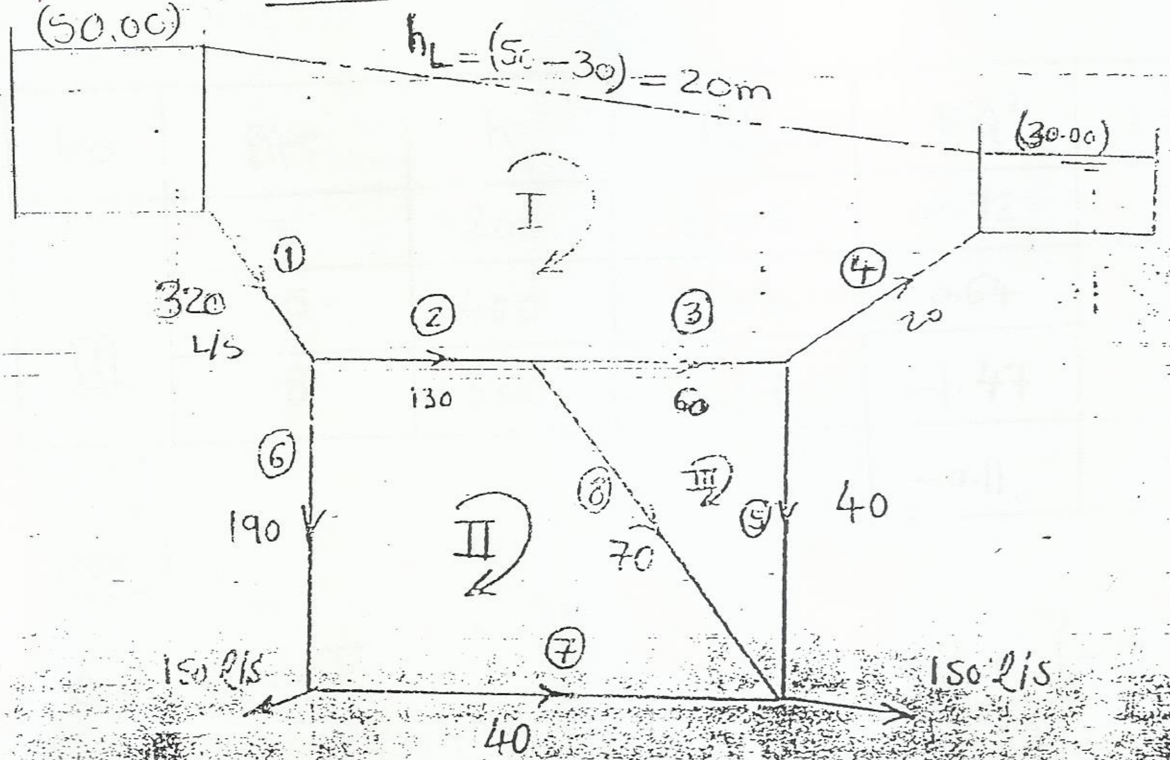
⇒ get ($d_{eq}$)

* **For loop (I):**




$$\sum h_L = 0 \Rightarrow -K_1Q_1^n + K_2Q_2^n + K_3Q_3^n = \text{Zero}$$

Ex: 5




| loop | pipe | K | Q m³/s | $KQ^2$ | 2\|KQ\| |
|---|---|---|---|---|---|
| I | 1 | 100 | −0.32 | −10.24 | 64 |
| | 2 | 500 | −0.13 | −8.45 | 130 |
| | 3 | 200 | −0.06 | −0.72 | 24 |
| | 4 | 100 | −0.02 | −0.04 | 4 |
| | | | Σ | −19.45 | 222 |

| loop | pipe | K | Q m³/s | $KQ^2$ | 2\|KQ\| |
|---|---|---|---|---|---|
| II | 2 | 500 | 0.13 | 8.45 | 130 |
| | 8 | 300 | 0.07 | 1.47 | 42 |
| | 7 | 400 | −0.04 | −0.64 | 32 |
| | 6 | 300 | −0.19 | −10.83 | 114 |
| | | | Σ | −1.55 | 318 |

| loop | Pipe | K | $Q_{m^3/s}$ | $KQ^2$ | $2\|KQ\|$ |
|---|---|---|---|---|---|
| III | 3 | 200 | 0.06 | 0.72 | 24 |
| | 5 | 400 | 0.04 | 0.64 | 32 |
| | 8 | 300 | -0.07 | -1.47 | 42 |
| | | | | -0.11 | 98 |

loop I

$$\Delta Q_1 = \frac{-\sum KQ^n}{n\sum|KQ^{n-1}|} = -\frac{\left[(50-30)+(-19.45)\right]}{222}$$

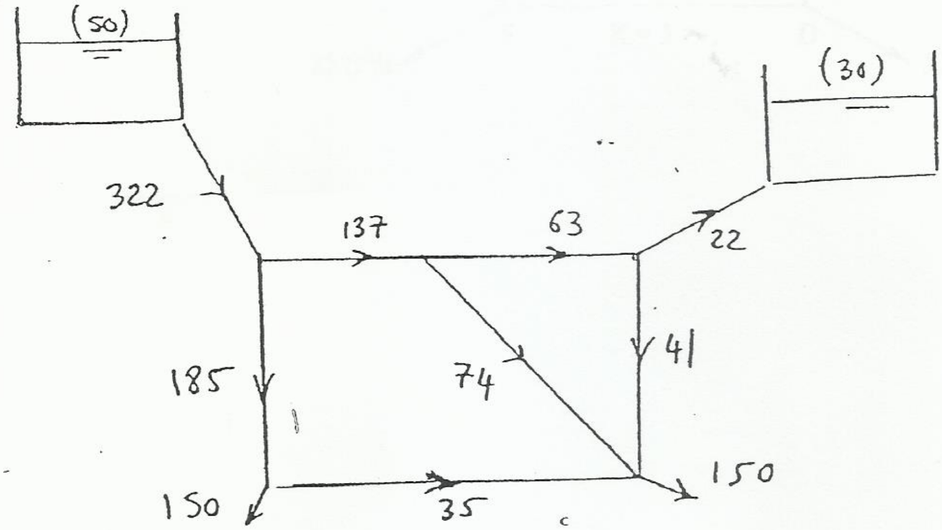
$$\Delta Q_1 = -2.48 \times 10^{-3}\ m^3/s = -2.48\ l/s$$

loop II

$$\Delta Q_{II} = \frac{-(-1.55)}{318} = 0.0049\ m^3/s$$

loop III

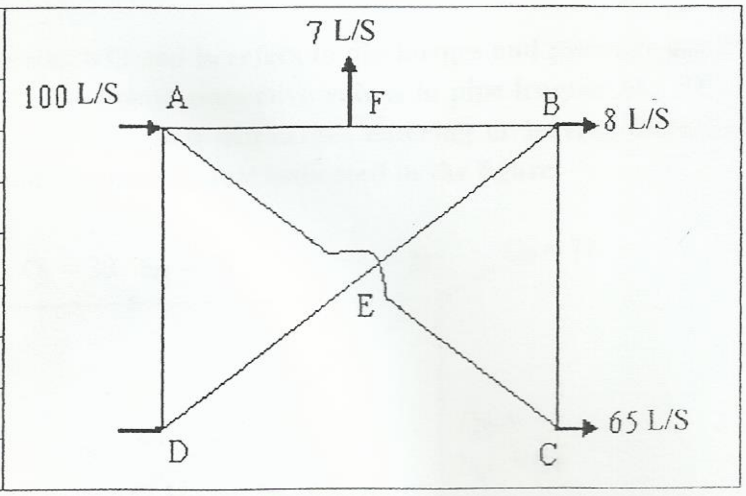
$$\Delta Q_{III} = 0.001\ m^3/s$$



4) Using two steps of hardy-cross method, and assuming F=0.02 for all pipes

a) Estimate the flow rate in each of the pipes in the network shown in figure.
b) If the elevation at point A is 100 m and the pressure head is 5 bars and the elevation at B is 90 m, find the pressure head at B in bars. Neglect velocity head.

| Pipe | Diam (mm) | Length (m) |
|---|---|---|
| AF | 300 | 400 |
| FB | 300 | 400 |
| BC | 300 | 600 |
| CE | 300 | 500 |
| EA | 300 | 500 |
| AD | 300 | 600 |
| DE | 300 | 500 |
| EB | 300 | 500 |

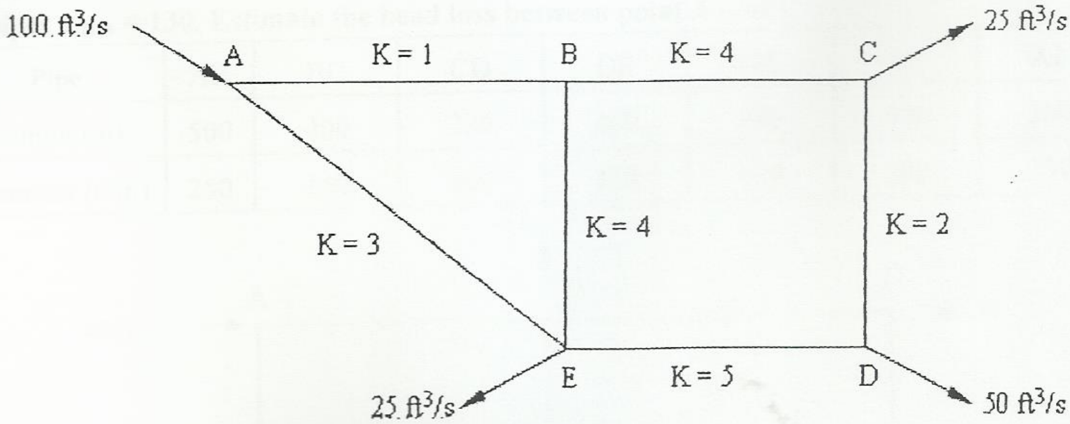



5) **Mandatory for all sections**

**For the pipe network shown in Fig, write down the equations required to solve the analysis problem using the following variables:**

**a) Pipe flows (Q-system)**

**b) Heads at junctions (H-system)**



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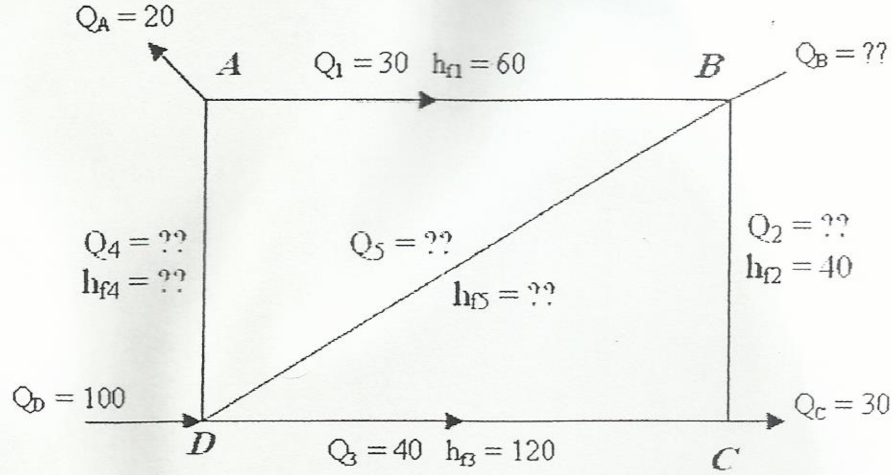
# ASSIGNMENT NO. (8)
## Pipe Networks

1) What do you understand from the term reliability in pipe networking systems, Give an example?

   Classify the different analysis techniques used in design of pipe networks

2) **A pipe network is shown in figure in which Q and $h_f$ refers to discharges and pressure head loss respectively. Subscripts 1, 2, 3, 4 and 5 designate respective values in pipe lengths AC, BC, CD, DA, and AC. Subscripts A, B, C, and D designate discharges entering or leaving the junction points A, B, C and D respectively. Find the missing data indicated in the figure.**




3) **In the shown network, a valve in BC is partially closed and the $K_{valve} = 10$. Using two steps of Hardy-Cross method, Find the flow in each pipe if Hazen-William's conveyance coefficient for all pipes $C_{HW} = 130$. Estimate the head loss between point A and C.**

| Pipe | AB | BC | CD | DE | BE | EF | AF |
|---|---|---|---|---|---|---|---|
| Length (m) | 500 | 400 | 200 | 400 | 200 | 600 | 300 |
| Diameter (mm) | 250 | 150 | 100 | 150 | 150 | 200 | 250 |